MULTISTABILITY IN NETWORKS OF PLASTICALLY INTERACTING OSCILLATORS

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Real-world networks often consist of connected active elements and change their structure over time. Typically, links between network elements are reorganized in response to changes in the states of the elements. One of the most intriguing systems that can reconfigure its connections is a neural network.

Spike-timing-dependent plasticity (STDP) refers to a concept of neural network formation in which the relative timing between the spikes of interacting neurons dictates the modulation of synaptic junctions strength. One of the most commonly used STDP rules describes discontinuous excitatory synapse. Here, the neuron spike is followed by two links-changing processes: (i) the strength of all connections that direct to spiking neuron are increased; (ii) the strength of the links going from spiking to other neurons is decreased. The change of the synaptic weight exponentially decreases as the time interval between the two nearest spikes of the interacting neurons increases. It is assumed that such activity-dependent changes in synaptic transmission provide a neural basis for the realization of higher brain functions, such as learning and memory.

Modeling plastic neural networks is a complex numerical task for two reasons: (i) neural dynamics and adaptation of synaptic weights usually occur at different time scales; (ii) the number of dynamic variables associated with the slow adaptation of synaptic weights increases quadratically with the size of the network. Because of these difficulties, various simplifications are used. One way is to implement networks with simplified neural models as well as simplified STDP rules.

As an example, here we consider Kuramoto-type networks with coupling weights dynamics governed by phase-difference-dependent plasticity (PDDP) rule, first introduced in Ref. [1]. The PDDP rule is built similarly to the STDP rule, but instead of the time difference, the phase difference of the oscillators is used. Moreover, the instantaneous change in synaptic weights is replaced by their continuous change in time. The relationship between PDDP and STDP is obtained by averaging STDP over time. PDDP has an advantage over STDP because it is more amenable to analytical analysis. We investigate the PDDP effect on the formation of a star-type network. In the star network, there is a center node (hub), and each of the other nodes (leaves) is connected only to this center but not between each other. The star network can be considered as an essential building block in real neural networks; it is the simplest network model that captures the sparse, clustering, small-world, and other important properties of many real-world networks. An advantage of a star network over more complex networks is that the number of dynamic variables associated with synaptic weights increases linearly, rather than quadratically, with the size of the network. Here we analyze the multistable dynamics of such networks caused by PDDP.

We show that a network with N leaves can evolve into 2N various asymptotic states, characterized by different values of the coupling strength between the hub and the leaves. Starting from the simple case of two coupled oscillators, we develop an analytical approach based on two small parameters ε and μ, where ε is the ratio of the time scales of the phase variables and synaptic weights, and μ defines the sharpness of the plasticity boundary function. The limit $\mu \to 0$ corresponds to a hard boundary. The analytical results obtained on the model of two oscillators are generalized for multi-leaf star networks. Multistability with 2N various asymptotic states is numerically demonstrated for one-, two-, three- and nine-leaf star-type networks. The results are published in Ref.[2].

References

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2. I. Ratas, K. Pyragas & P. A. Tass, Multistability in a star network of Kuramoto-type oscillators with synaptic plasticity, Sci. Rep. 11, 9840 (2021)